Rheological Properties of Hydroxypropyl Guar (HPG) Slurries

Subhash N. Shah
Halliburton Services, P.O. Drawer 1431, Duncan, OK 73536

Rheological behavior of moderately concentrated (up to 35 vol. %) suspensions or slurries prepared with non-Newtonian carrier fluids is investigated. Extensive experimental data on the Poiseuille slit flow of suspensions of sand in Hydroxypropyl Guar (HPG) solutions are presented. Flow data are gathered by pumping various non-Newtonian slurries into a unique, 8 ft (2.4 m) tall vertical apparatus, with great care taken to calibrate the instrument and to avoid settling difficulties. Investigated are the effects of polymer concentration, solids concentration, test temperature, and shear rate on the rheological properties of slurries. The rheological behavior of HPG solutions and HPG suspensions is characterized by a pseudoplastic power law non-Newtonian fluid model over a shear rate range of 30 to 500 s^{-1}. The correlations for predicting the relative slurry viscosity for HPG fluids are presented. Substantial increase in viscosity of these non-Newtonian fluids is observed, and they are found to be a strong function of polymer concentration, shear rate, test temperature, and particle concentration. Results of this investigation are also compared with those available for Newtonian and non-Newtonian suspensions.

Introduction

There exist numerous correlations/equations and experimental data to estimate the increase in viscosity of a Newtonian carrier fluid (or relative slurry viscosity) by addition of solids. By way of contrast, very few studies have dealt with the flow behavior of suspensions or slurries prepared with non-Newtonian carrier fluids.

Einstein (1905, 1906) first determined theoretically that infinitely dilute suspensions of monomodal rigid spheres behave as a Newtonian fluid and proposed a simple expression for the relative viscosity as a function of volume fraction solids. Subsequently, many investigators have developed equations extending the simple Einstein expression to highly concentrated slurries by incorporating higher order terms (Eilers, 1941; Mooney, 1951; Rutgers, 1962; Thomas, 1965; Landel et al., 1965; Frankel and Acrivos, 1967; Chong et al., 1971; Yao and Matsumoto, 1987).

It has been known that the slurries prepared with the Newtonian carrier fluids can exhibit a Newtonian behavior at low volume fraction of solids but may exhibit non-Newtonian behavior at high solids concentrations (Ackerman and Shen, 1979; Dabak and Yucel, 1986; Satchwell et al., 1988; Tsai et al., 1989; Agarwala et al., 1990). This means that the viscosity equation must incorporate the shear rate effect (Govier, 1972; Govier et al., 1972; Tsutsumi and Yoshida, 1987). Above a threshold volume fraction, the rheological behavior of many Newtonian slurries can be either described by the pseudoplastic type model or Bingham plastic model.

Rheological correlations for non-Newtonian slurries, that is, the suspensions in non-Newtonian carrier fluids, have met with limited success (Nicodemo and Nicolais, 1974; Quemada, 1978).

Faulkner and Schmidt (1977) rheologically characterized glass bead-filled polypropylene composites in the shear rate range of 50 to 600 s^{-1} using a capillary rheometer and reported that the power law exponent decreases as the volume fraction of beads increases. Further, the relative viscosity decreases with increasing shear rate for the volume fraction range of 0 to 0.26. Sundstrom (1983) extended the Kataoka et al. (1978) simple correlation of viscosities of filled polymers to suspensions of particles in polymeric solutions. Relative viscosities defined at the same shear stress have been correlated with volume fraction by the one-constant equation of Maron and Pierce (1956). Tanaka and White (1980) attempted to extend the analysis of Frankel and Acrivos to the power law fluid. Later, Jarzebski (1981) reported that as the flow behavior index n of the power law model approaches 0.5, the Tanaka and White (1980) analysis breaks down. He developed an expression that allows one to predict the dependence of a concentrated pseudoplastic suspension's shear viscosity on concentration for all values of n. The suspension was, however, assumed to have the same power law index as that of the fluid in the suspension. For a concentrated power law suspension use of either the modified Frankel and Acrivos equation or modified Mooney's equation is suggested. Chan and Powell (1984) presented the results of steady shear experiments as well as oscillatory experiments. It is reported that for the non-Newtonian fluid, the steady shear viscosity increases with increasing volume fraction at all shear rates, the increase being greater as shear rate approaches zero. Recently, Poslinski et al. (1988) have investigated the influence of particulate and matrix properties as well as bimodal size distribution of particulates on the shear viscosity, primary normal stress coefficient, dynamic viscosity, and storage modulus of filled polymeric systems.

Metzner (1985) has provided an excellent review of the flow behavior of suspensions as may be of interest in the processing of composites and similar polymeric materials. He described the overall state of the art as good but has identified several remaining problems and research areas.

For non-Newtonian slurries, Nolte (1988) has recommended the use of modified Landel's equation for Newtonian suspensions. The modified Landel's equation is independent of shear rate. Results of tests conducted in a specially designed Couette viscometer with HPG fluids containing 18-25 mesh neutrally buoyant beads have been reported. Keck et al. (1989) studied the effect of particle concentration on the effective viscosity of non-Newtonian fluids and presented a modified Eiler's expression which includes the effect of shear rate, temperature, polymer concentration, and particle concentration. Tests were conducted employing a rotational viscometer with HPG fluids containing neutrally buoyant 60–100 mesh styrene divinylbenzene beads. Data were gathered only at three shear rates, viz. 5, 170, and 1,000 s^{-1}.

It is very apparent from the above literature review that the state-of-the-art research work for non-Newtonian slurries is limited. Further, systematic research work is definitely needed. In this article, we address some of the open questions and compare the findings with the available information from the literature.

Equipment and Procedure

A schematic diagram of the experimental setup designed to investigate the rheological behavior of non-Newtonian slurries is depicted in Figure 1. It consisted of a vertical slot flow model, fluid mixing tank, centrifugal pump, Moyno® pump, magnetic flowmeter, mass flowmeter, double pipe heat exchanger, temperature probes, and differential pressure transducer.

The slot model consisted of two parallel sheets of Plexiglas® with dimensions 2.44 m long × 25.4 cm wide × 1.3 cm thick. These sheets were separated by 1.3 cm gap using 1.3 cm × 2.54 cm spacers. Fluid was pumped into the test model through the bottom from a slotted horizontal pipe attached to the model. The fluid exited the model through two openings in one face of the model into the mixing tank.

To monitor differential pressures at various fluid flow rates, two pressure ports were drilled: one 61 cm above the model entrance and the other one 30.5 cm below the exit. This arrangement allowed 1.52 m between pressure ports for the dif-

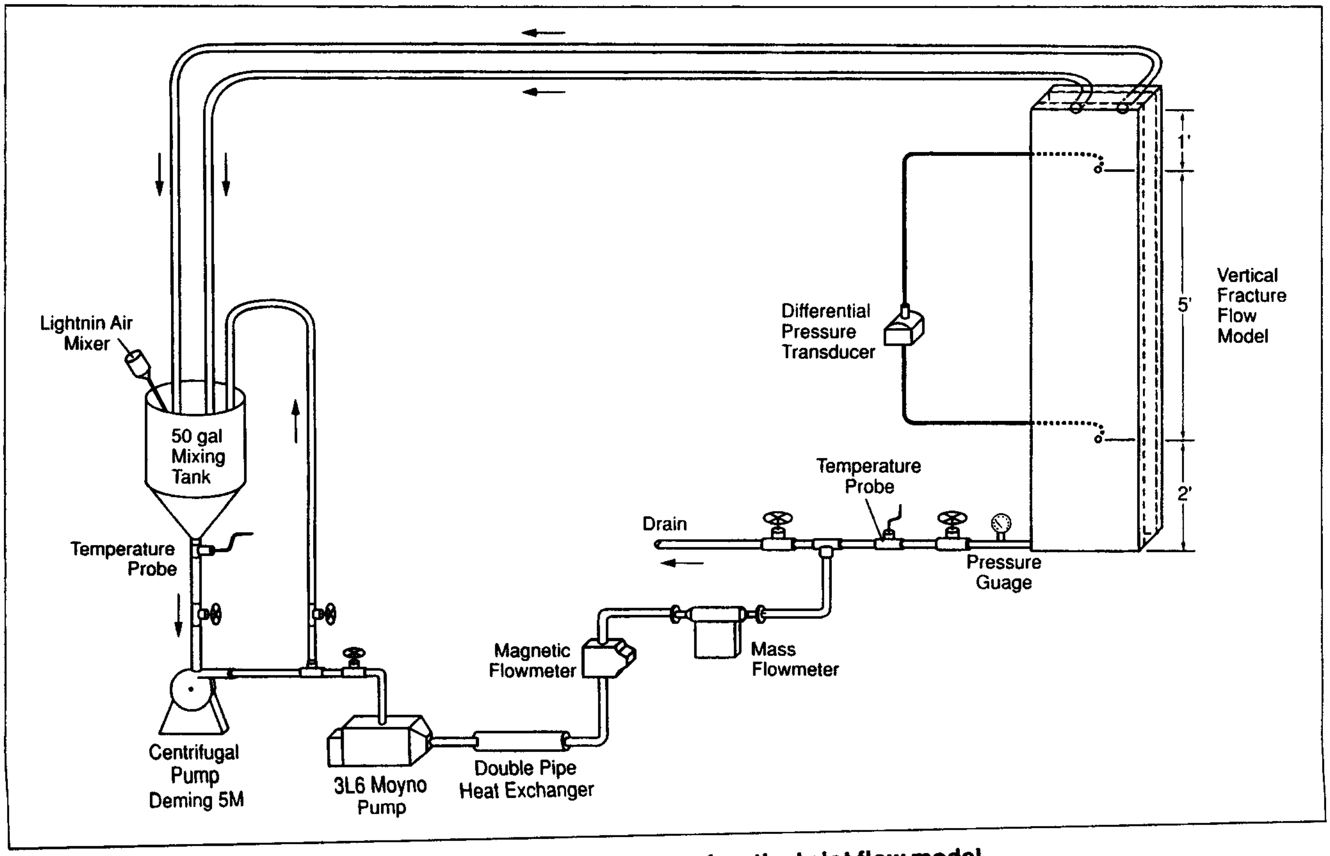

Figure 1. Experimental setup of vertical slot flow model.

Table 1. Typical Particle Size Distribution (20-40 Mesh Sand)

Typical Screen Analysis	% Retained
+16	0
−16 +20	0.77
−20 +30	33.92
−30 +35	42.24
−35 +40	16.77
−40 +50	6.24
−50 +Pan	0.05

Sphericity = 0.8
Specific gravity = 2.65

ferential measurements. A 2.5 psid (17.2 kPa) pressure transducer manufactured by Statham was used to monitor differential pressures. The transducer was precisely calibrated with standard dead weight calibration and found to be well within an accuracy of ±0.5% of the pressure reading.

To monitor the slurry rate and to compute the slurry concentration, a 2.54 cm Foxboro magnetic flowmeter and MicroMotion flowmeter were installed in the flow loop. Two temperature probes, one in the mixing tank and the other in the pipe leading to the entrance of the model, were also installed to monitor the fluid temperature.

To help insure uniform slurry concentration throughout the test, the 189.3 dm^3 stainless steel mixing tank was equipped with an air driven mixer. A Deming 5M centrifugal pump was also connected to the mixing tank. A double pipe heat exchanger was used to control the fluid temperature. Each instrument used in this study was precisely calibrated.

Fluids selected for testing included 3.6, 4.8, and 7.2 kg/m^3 HPG (Rhone-Poulenc, Inc., molecular weight 1 to 2 million). The solid particles used were 20-40 U.S. Mesh sand (see Table 1) and concentrations varied from 0 to 0.35 volume fraction. The effect of three test temperatures, that is, 26.7, 43.3, and 60.0°C was also investigated.

To prepare aqueous based fluids, first the HPG polymer at desired concentration was added to the fresh water through an eductor while running the centrifugal pump. Immediately following polymer addition, buffers were added to achieve the desired pH. The polymer solution was then hydrated for 15 min. while circulating through the test cell.

The data on the steady-state differential pressure vs. flow rate were recorded first with fluid without solids. A sample of HPG solution was also taken from the mixing tank and its rheological measurements were recorded using a model 35 Fann rotational viscometer. This viscometer is a multispeed rotational viscometer with outer cylinder (36.83 mm ID, 87 mm long) rotating and inner cylinder (34.49 mm OD, 38 mm long) stationary. Then sand was slowly added to obtain a homogeneous slurry of 0.084 volume fraction while the fluid was circulated through the flow loop. Flow data were recorded and additional sand was added for the next concentration, that is, 0.15 volume fraction. This procedure was repeated for 0.21, 0.27, 0.31, and 0.35 volume fractions.

Results and Discussion

Model width measurements

Pipe flow measurements are very sensitive to the internal diameter of the pipe; likewise slot flow measurements are also very sensitive to width of the slot.

To determine an average width of the slot, a 65% sugar solution (Newtonian calibration fluid) was prepared at 19.4°C and differential pressure vs. flow rate data were acquired. While gathering slot flow data with sugar solution, a sample of the same fluid was tested on a model 35 Fann viscometer. From these measurements, the hydrodynamic average width of the model was calculated to be 1.270 cm. In subsequent data analysis, this model width was used.

Another approach taken to measure the average width of the slot model was to use a micrometer to take measurements at three different places along the model height. With the exact thickness of the two parallel acrylic panels known, an average inside width of 1.265 cm was calculated. Thus, this average width agreed very well with the average width determined from the experimental sugar solution data. Micrometer measurements were also conducted while flowing slurries through the model and no change in the readings was noticed.

As mentioned earlier, tests were also run at 43.3 and 60.0°C. It is reasonable to assume that for the tests at 60.0°C, the model width may have been altered. Therefore, it was decided to gather flow data with sugar solution at 60.0°C. The model flow data were compared with the data obtained from the model 35 Fann viscometer with the sample taken simultaneously. Both data sets were in excellent agreement indicating no appreciable change in the original width determination from the 26.7°C data with sugar solution. Again, it is not surprising since the heat distortion temperature for acrylic is in the range of 65.6 to 98.9°C.

Rheological properties of slurries

Since the polymer solutions and suspensions exhibited pseudoplastic non-Newtonian type behavior, they could be characterized by Ostwald-de Waele or power law fluid model.

$$\tau = k\left(-\frac{d\bar{v}}{dw}\right)^n \tag{1}$$

In terms of wall shear stress, $\tau_w = (w\Delta p/2L)$ and wall shear rate, $[(-d\bar{v}/dw)_w]$, Eq. 1 can be rewritten as

$$\tau_w = k\left(-\frac{d\bar{v}}{dw}\right)_w^n \tag{2}$$

The Rabinowitsch-Mooney type analysis was performed with all the experimental data obtained in this study. The differential pressure and flow rate data of all fluids with and without solids were converted to wall shear stress, τ_w and nominal shear rate, $\gamma = (6\bar{v}/w)$. As suggested by Metzner and Reed (1955), a logarithmic plot of τ_w vs. γ was then prepared for each case. For all fluids a straight line was obtained. Therefore, both n' and K', the slope and unity intercept, respectively, of the tangent lines remained constant (that is, they were independent of τ_w). Thus, the following equation is the equation of the logarithmic curve as well as of its tangent.

$$\tau_w = K\left(\frac{6\bar{v}}{w}\right)^n \tag{3}$$

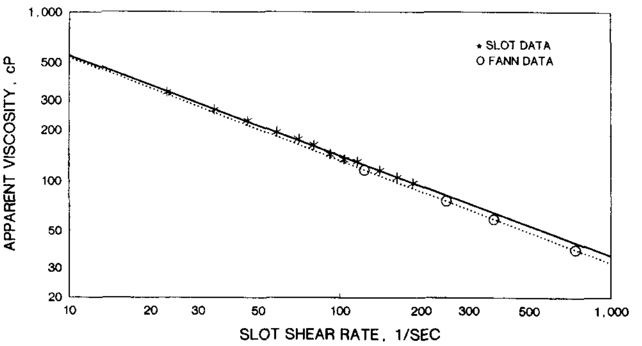

Figure 2. Comparison of slot data with rotational viscometer data.

7.2 kg/m³ HPG solution at 60.0°C.

The usual point-by-point evaluation of the slope of the curve was, therefore, not required and $n' = n$ and $K' = K$.
The k in Eq. 2 and K in Eq. 3 are related to each other by the following:

$$k = K\left(\frac{3n}{2n+1}\right)^n \tag{4}$$

An apparent viscosity, μ_a of solution as well as suspensions were calculated by dividing wall shear stress by the corresponding nominal slot shear rate. Thus,

$$\mu_a = \frac{\tau_w}{\gamma} = K(\gamma)^{n-1} \tag{5}$$

A logarithmic plot of the apparent viscosity vs. slot shear rate was then prepared for each fluid.

With each fluid without sand, Fann viscometer data were also recorded. The data gathered with this instrument were converted to obtain an apparent viscosity and slot shear rate and then they were compared with the model data. Figure 2 depicts such a comparison of the viscosity data taken from the slot flow with those taken with the rotational viscometer for 7.2 kg/m³ HPG solution at 60.0°C. An excellent agreement is seen between both data sets.

Figure 3 presents the apparent viscosity vs. slot shear rate data of 7.2 kg/m³ HPG fluid with and without sand at 60.0°C. Though not included in the article, the data of other tested fluids were also plotted in this fashion.

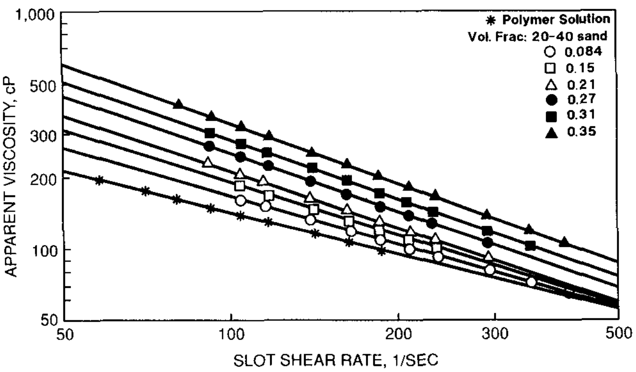

Figure 3. Viscosity vs. slot shear rate.

7.2 kg/m³ HPG slurries at 60.0°C.

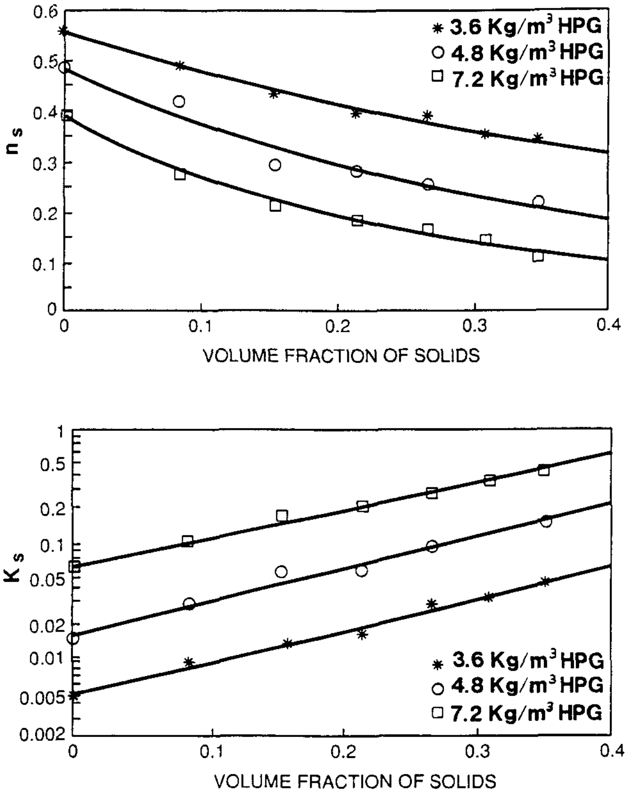

Figure 4. n_s, K_s vs. volume fraction of solids 43.3°C.

Several observations can be made from the data presented in Figure 3. First of all, for the shear rate range studied, a linear relationship between apparent viscosity and shear rate on a logarithmic paper with all fluids with and without sand is evident. This means that the rheological behavior of all fluids, HPG solutions as well as slurries, can adequately be described by the power law expression given by Eq. 3. Second, a significant increase in an apparent viscosity of HPG solution is seen because of the addition of sand. Third, the viscosity increase is more pronounced at lower shear rates than at higher shear rates.

The experimental data of an apparent viscosity vs. shear rate as depicted in Figure 3 were fitted with the least-squares curve fit technique, and the power law fluid parameters, n and K for solutions as well as n_s and K_s for slurries were evaluated. Thus, n and K for all solutions and n_s and K_s for all slurries tested in this investigation were determined.

The n_s and K_s values thus determined are plotted as a function of volume fraction solids in Figure 4 for all fluids tested at 43.3°C. The values at zero volume fraction solids represent the corresponding HPG solution values. Similar plots of all fluids tested at 26.7°C and 60.0°C were also prepared. The trends of n_s and K_s with volume fraction solids at other temperatures were all the same but higher n_s and lower K_s values as temperature increased were noticed which was the direct result of the thermal thinning of the polymer solutions at higher temperatures.

The n_s and K_s vs. volume fraction solids data in Figure 4 show that for all fluids, as volume fraction of solids increases, the n_s values decrease while the K_s values increase. This means that the degree of non-Newtonian character of the HPG so-

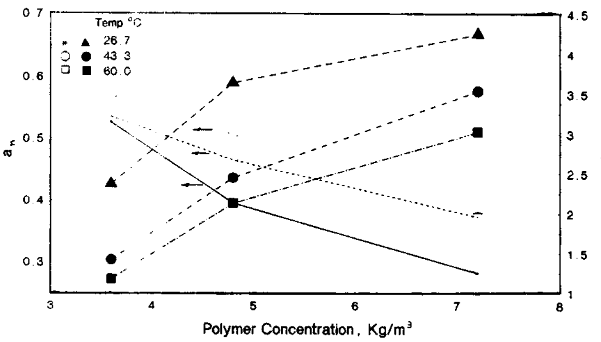

Figure 5. Parameters a_η and b_η vs. polymer concentration.

lutions increases by adding sand to them. Some investigators (Tanaka and White, 1980; Jarzebski, 1981; and Nolte, 1988) in the past have assumed that the n_s values of slurries remain the same as the carrier fluid n. This is simply not true for the HPG fluids studied in this investigation. The dramatic effect on n_s and K_s produced by increasing solids content is seen in Figure 4. The n_s values, however, seem to level off or reach an asymptotic values at higher solids concentration. One can, however, obtain the same n value for slurry and carrier fluid by selecting data from a narrow shear rate range in Figure 3 and approximating them as parallel lines.

Regression analysis was performed on n_s and K_s data of Figure 4 employing the equations of the following form:

$$n_s = a_n \cdot (\exp) b_n \phi \tag{6}$$

$$K_s = a_k \cdot (\exp) b_k \phi \tag{7}$$

where a_n, b_n, a_k, and b_k are constants. The Eqs. 6 and 7 reduce to the HPG solution for $\phi = 0$.

The variations of constants a_n, a_k and exponents b_n, b_k in Eqs. 6 and 7 as a function of polymer concentration and test temperature are depicted in Figures 5 and 6.

From the regression data, a relative viscosity which is the ratio of an apparent viscosity of slurry to that of solution, was calculated as a function of shear rate for each test. The sand concentration was converted to volume fraction of solids to facilitate the comparison of experimental results with the published literature.

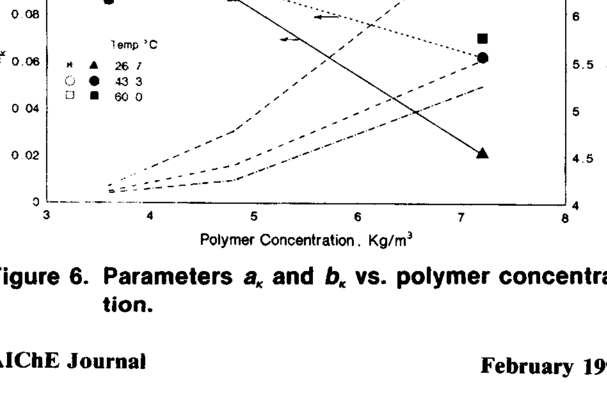

Figure 6. Parameters a_κ and b_κ vs. polymer concentration.

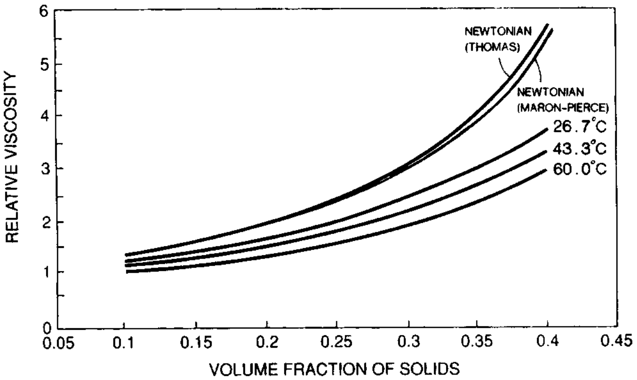

Figure 7. Effect of sand concentration on relative viscosity.

4.8 kg/m^3 HPG; shear rate = 100 1/s.

Figure 7 depicts the effect of volume fraction of solids on the relative viscosity for 4.8 kg/m^3 HPG fluid at 100 s^{-1} and at test temperatures of 26.7, 43.3, and 60.0°C. Newtonian relative viscosity-concentration curves based on Thomas (1965) and Maron and Pierce (1956) correlations are also shown for comparison purposes. It should be noted that both Thomas and Maron-Pierce equations agree very well with each other for the solids concentration range shown. Even at a moderately high solids concentration of 0.4, both equations predict relative viscosities within 3.4%.

It can be seen from Figure 7 that the relative viscosity increases gradually at lower sand concentrations but increases very rapidly at higher sand concentrations. At low particle concentration, the hydrodynamic interaction is a dominating factor while at higher particle concentration, interparticle interaction becomes a dominating factor.

Furthermore, in Figure 7 relative viscosities of all non-Newtonian slurries at all temperatures are substantially lower than those predicted by either the Thomas equation or Maron-Pierce equation for Newtonian slurries. At low sand concentration the error in using a Newtonian equation for estimating relative viscosity of a non-Newtonian slurry is not very much but as particle concentration increases the deviation between the two becomes very significant. Large errors may be committed by not using proper equations for non-Newtonian slurries. The rapid increase in relative viscosity at higher concentration of solids is because of the denser packing and arrangement of particles.

The effect of polymer concentration on the relative viscosity is shown in Figure 8. The relative viscosity vs. volume fraction of solids data for 3.6, 4.8, and 7.2 kg/m^3 HPG fluids at 43.3°C and 100 s^{-1} are depicted. As the polymer concentration increases (power law exponent n decreases, that is, the viscosity of the carrier fluid or suspending liquid increases), the relative viscosity of the pseudoplastic slurry decreases. For Newtonian slurries, on the other hand, the relative viscosity is only slightly affected by the carrier fluid viscosity. From the results shown in Figure 8 it is important to know that the more viscous the fluid is (lower n value), its relative viscosity will deviate further from the Newtonian predictions.

As described earlier, some investigators in the past have ignored the effect of shear rate on the relative viscosity of pseudoplastic non-Newtonian slurries. From results shown in Figure 9 for 4.8 kg/m^3 HPG fluid at 60°C, it can be seen that

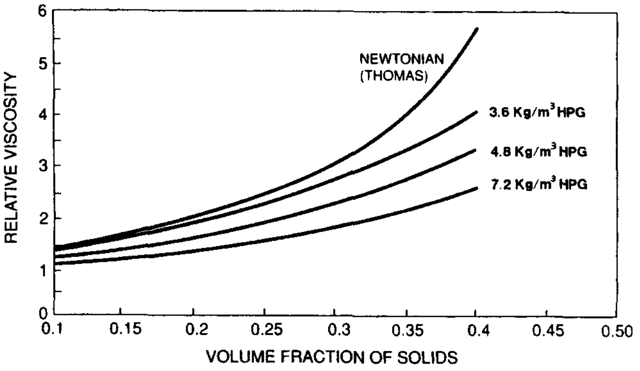

Figure 8. Effect of polymer concentration.
43.3°C; shear rate = 100 1/s.

there is a significant effect of shear rate on the relative viscosity of non-Newtonian slurries. Relative viscosity decreases, as the shear rate increases. Again, the effect is more pronounced at higher solid concentration than at low solid concentration.

Next, the influence of temperature on the relative viscosity of non-Newtonian slurries is examined. Figure 10 presents the results of 3.6, 4.8, and 7.2 kg/m^3 HPG fluids with 0.3 volume fraction solids and at a shear rate of 50 s^{-1}. It can be seen from Figure 10 that temperature also has a dramatic effect on the relative viscosity of slurries. This is partly because of the reduction in carrier fluid viscosity due to thermal effects. The relative viscosity increases as the temperature increases.

Comparison with previous studies

Experimental results presented in this study were procured for the purpose of knowing the increase in viscosity produced by the addition of sand to fluids generally used during a hydraulic fracturing treatment of an oil or gas well. A fluid containing sand is pumped under high pressure to break open the formation rock underground and to keep open the created fracture after pumping has ceased. A 20-40 U.S. mesh size sand is the most commonly used particulate for hydraulic fracturing and it was selected for this investigation. It is recognized that 20-40 sand is not very spherical and thus, comparison of the results of this study with earlier work dealing with suspensions of spherical particles is kept at a minimum. The predictions of relative viscosity at high sand concentration ($\phi = 0.35$) from this study are compared with the Jarzebski (1981) analysis for concentrated pseudoplastic slurries and with the modified Frankel and Acrivos (1967) equation as suggested by Jarzebski. The predicted results at low sand concentration ($\phi = 0.15$) are compared with the modified Frankel and Acrivos.

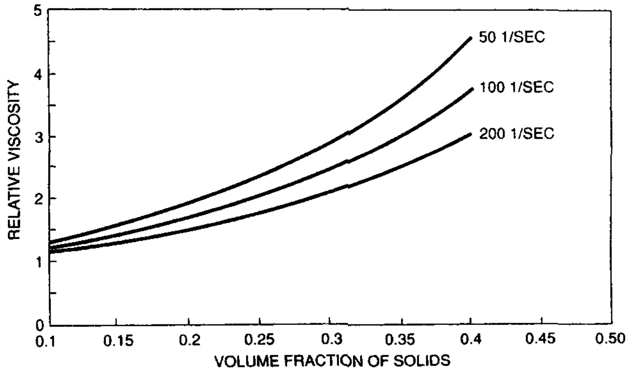

Figure 9. Effect of shear rate on relative viscosity.
4.8 kg/m^3 HPG at 60.0°C.

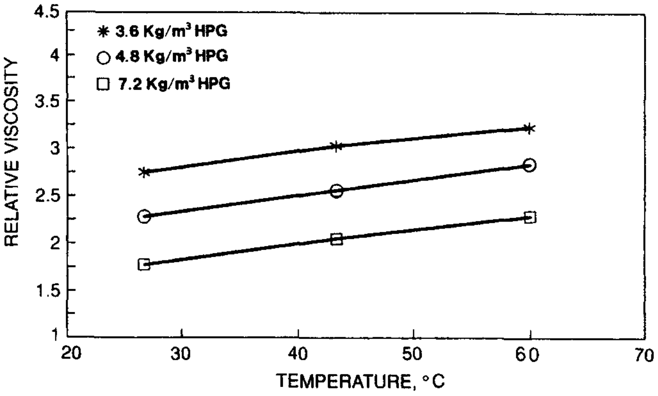

Figure 10. Effect of temperature on relative viscosity.
Shear rate = 50 1/s; volume fraction = 0.3.

Table 2 compares the predicted relative viscosities from this study at $\phi = 0.35$ and at shear rates of 50, 100, and 200 s^{-1} with those predicted from the Jarzebski and modified Frankel-Acrivos equations. It should be noted here that both the Jarzebski and modified Frankel-Acrivos equations are independent of shear rate while the results of the present study are dependent on shear rates as shown in the table. The experimental results match reasonably well with the Jarzebski's analysis at combinations of low n and shear rate values and also at combinations of high n and shear rate values. The experimental results for the shear rate of 100 s^{-1} match fairly good with the modified Frankel-Acrivos predictions.

A similar comparison of experimental results with the modified Frankel-Acrivos equation for $\phi = 0.15$ is presented in Table 3. Reasonable agreement is obtained only at shear rate of 50 s^{-1} and at higher n values. At higher shear rates, the modified Frankel-Acrivos equation overpredicts the relative viscosities significantly.

Figure 11 presents a plot of relative viscosity of all HPG fluids tested at 26.7°C and shear rate of 170 s^{-1} as a function of volume fraction particles. The Newtonian curve based on the Thomas equation is again shown for comparison. The dotted curves are our predictions from the correlations developed in this study while the points are the published data of Keck (1989). The shear rate of 170 s^{-1} and test temperature of 26.7°C were chosen so that our experimental data can be directly compared with Keck's data.

Results of Figure 11 show that our relative viscosities, in general, are slightly lower than Keck's relative viscosities. As mentioned earlier, Keck used neutrally-buoyant 60-100 mesh (250-150 μm) styrene divinylbenzene beads with specific gravity of 1.05, which are much smaller in size than the 20-40 mesh (700 μm) sand particles used in this research. Smaller particle size will make his slurries more viscous than the slurries reported in this work. Therefore, Keck's relative viscosity values should be higher than our relative viscosity values which is the case. This finding is also in agreement with the previously reported results of other investigators and a recent publication of Agarwala et al. (1990) in which it is shown that as particle

Table 2. Comparison of Experimental Results With Theory For $\phi = 0.35$

Carrier Fluid kg/m³	Temp. °C	Carrier Fluid n	Shear Rate, s^{-1}			Jarzebski	Modified Frankel-Acrivos
			50	100	200		
3.6	26.7	0.526	3.51	2.86	2.33	2.85	3.12
4.8	26.7	0.396	2.85	2.35	1.93	2.67	2.42
7.2	26.7	0.283	2.07	1.78	1.53	2.19	1.95
3.6	43.3	0.536	3.80	3.29	2.84	2.91	3.18
4.8	43.3	0.465	3.20	2.65	2.20	2.72	2.77
7.2	43.3	0.373	2.48	2.06	1.72	2.44	2.32
3.6	60.0	0.571	4.07	3.56	3.12	3.03	3.40
4.8	60.0	0.506	3.58	2.98	2.48	2.81	3.00
7.2	60.0	0.387	2.79	2.34	1.96	2.60	2.38

Table 3. Comparison of Experimental Results With Theory For $\phi = 0.15$

Carrier Fluid, kg/m³	Temp. °C	Carrier Fluid n	Shear Rate, s^{-1}			Modified Frankel-Acrivos
			50	100	200	
3.6	26.7	0.526	1.52	1.36	1.22	1.59
4.8	26.7	0.396	1.32	1.17	1.05	1.46
7.2	26.7	0.283	1.17	1.07	1.01	1.36
3.6	43.3	0.536	1.69	1.57	1.46	1.60
4.8	43.3	0.465	1.47	1.33	1.21	1.53
7.2	43.3	0.373	1.26	1.13	1.02	1.44
3.6	60.0	0.571	1.76	1.65	1.55	1.64
4.8	60.0	0.506	1.57	1.43	1.30	1.57
7.2	60.0	0.387	1.36	1.24	1.12	1.45

size decreases, the relative viscosity increases. Further experiments will, however, be necessary with slurries prepared with various particle sizes in order to understand this complex phenomenon more thoroughly.

It should be stated here that overall our experimental results agree more closely with Keck's than with any other reported study.

Our experimental results are compared with the modified Landel's equation as suggested by Nolte (1988) in Figure 12 for 3.6 and 7.2 kg/m³ HPG fluids at 26.7°C and at 50 s^{-1}. Results of both studies show lower relative viscosities for non-Newtonian slurries than those of Newtonian slurries. As expected, at low volume fraction solids our results and Nolte's predictions match fairly close but as solids concentration increases more departure between the two results are seen. Our results in both cases show higher relative viscosities than Nolte's predictions. In other words, Nolte's predictions are lower than our experimental findings and they deviate more at lower polymer concentration and at higher sand concentration.

Furthermore, it should be remembered that unlike Nolte's proposed equation, our results show definite shear rate dependent relative viscosities. Also, the power law exponent, n_s of slurry is not the same as the carrier fluid n as assumed by Nolte. The decrease in μ_r with increasing shear rate at a constant ϕ is consistent with the behavior reported by Faulkner and Schmidt (1977) who investigated the effects of glass bead-filled polypropylene composites. Chan and Powell (1984) have also reported similar behavior with non-Newtonian suspensions. From Helical Screw Viscometer (HSV) data of 4.8 kg/m³ HPG solution containing 20% and 35% by volume solids, Prud'homme (1989) has also reported decreasing n values with increasing volume fraction solids.

Conclusions

- A vertical slot flow model has been utilized successfully to gather the flow data of 3.6, 4.8, and 7.2 kg/m³ HPG fluids containing 20-40 sand at a concentration of 0.084, 0.15, 0.21, 0.27, 0.31, and 0.35 volume fractions and at test temperatures of 26.7, 43.3, and 60.0°C.
- Correlations to predict the effect of sand on the apparent viscosity of HPG fluids as a function of polymer concentration, shear rate, test temperature, and sand concentration have been developed and presented.
- The addition of sand to the HPG solutions has shown

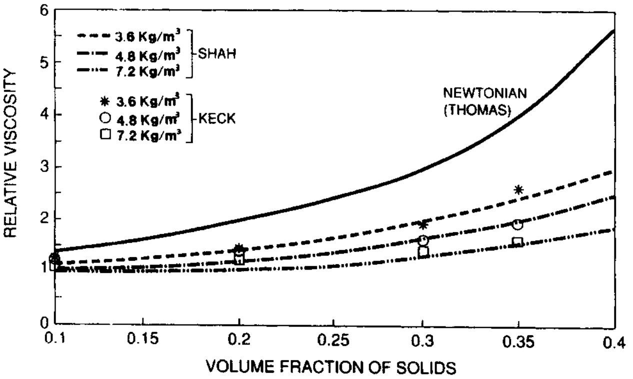

Figure 11. Comparison with Keck's data.

26.7°C; shear rate = 170 1/s.

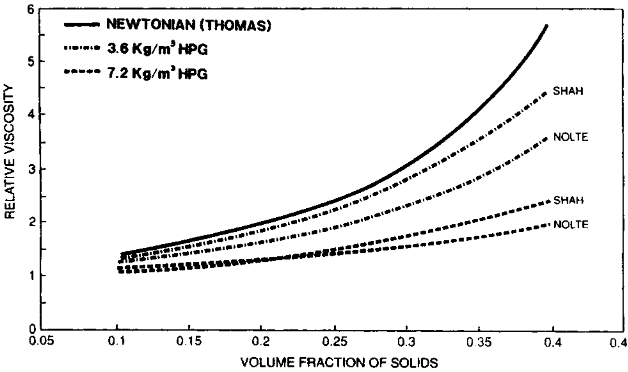

Figure 12. Comparison with Nolte's proposed equation.

26.7°C; shear rate = 50 1/s.

significant increases in their apparent viscosities. Relative viscosity is found to be a strong function of polymer concentration, shear rate, test temperature, and sand concentration.

- Higher relative viscosities are seen at lower polymer concentration, lower shear rate, higher solids concentration, and higher test temperature. All relative viscosities were, however, lower than those predicted for Newtonian fluids.
- Further research is required to investigate the effects of particle concentration on the relative viscosity of commonly used cross-linked (gel) non-Newtonian fluids. The effect of particle size and density on the relative viscosities of uncross-linked (solution) as well as cross-linked (gel) fluids should also be evaluated.

Notation

a, b = constants
K = consistency index, power law coefficient
K_s = slurry consistency index
L = slot length
n = flow behavior index, power law exponent
n_s = slurry flow behavior index
Δp = differential pressure
$\bar{v}$ = average fluid velocity
w = slot width

Greek letters

γ = slot shear rate, $6\bar{v}/w$
μ_a = carrier fluid apparent viscosity
μ_r = relative viscosity
μ_s = slurry viscosity
τ = slot shear stress, $w\Delta p/2\mathrm{L}$
ϕ = volume fraction of solids

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